Development and Design of a Signal Control System along the Route 2 in Hiroshima City

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Abstract
This paper deals with the study of a signal control system along the Route 2 in Hiroshima city from the viewpoint of a digital control for traffic flow dynamics. Time-varying characteristics of traffic flows are analyzed to evaluate the volume balance at each signalized intersection during the cycle length for each lane. The signal control system of the congestion length is described by a nonlinear time-varying discrete dynamic system and synthesized by using the feedback control based on the volume balance at each signalized intersection. The signal control algorithm in which the three signal control parameters are searched systematically so as to minimize the sum of congestion lengths along the arterials is presented. From the comparison of congestion lengths between measurement values controlled by a pattern selection method and simulation values, it is confirmed that the signal control system and the signal control algorithm work effectively along the Route 2 in Hiroshima city, Japan.

Keywords: traffic analysis, signal control system, signal control algorithm, simulation

1. INTRODUCTION
In recent years, the congestion has increased in urban traffic networks along with the increase of the number of vehicles registered in Japan. Traffic control is exercised through the combination of road markings, traffic signs and signals. The signal control is an effective method to control the congestion of traffic arterials. On-line signal control methods such as the SCOOT \(^1\), decentralized control \(^2\), MODERATOT \(^3\) and dynamic programming \(^4\) have been presented to control the congestion along arterials.

The common problem of the above-mentioned studies is as follows; the three signal control parameters are not controlled systematically but independently to minimize each performance criterion. Since incoming volumes and queue lengths at each signalized intersection increase rapidly in rush hours, the three signal control parameters are desired to be controlled systematically and cooperatively.

This paper studies the signal control of congestion lengths along two-way traffic arterials from a deterministic control viewpoint. The volume balance is held at each signalized intersection of the arterials for certain duration. Based on the volume balance at each signalized intersection the congestion mechanism is described quantitatively. A signal control algorithm is presented to control congestion lengths along arterials. From the comparison between measurement values controlled by a pattern selection method and simulation values, it is confirmed that the proposed control algorithm works well to control congestion lengths so as to minimize the performance criterion and to maximize the through band width along two-way traffic arterials.

2. TRAFFIC ANALYSIS

2.1 Volume balance
The signal control system of the congestion length is synthesized for two-way traffic arterials. The volume balance for each lane at each signalized intersection of the arterial is written as

\[
x_i(j, m, k) = x_i(j, m, k - 1) + x_i(j, m, k) - x_i(j, m, k)
\]

\[
\left\{
\begin{array}{l}
x_i(j, m, k) = \xi(j, m, k) \cdot \psi_i(j, m, k) \\
x_i(j, m, k) \geq 0
\end{array}
\right.
\]

where \( k = k \Delta T \) denotes time, \( j \) and \( m \) denote the location of each signalized intersection and the approach of vehicles respectively (see Fig. 1). \( x_i(j, m, k) \) and \( x_i(j, m, k) \) denote the excess incoming volume,
2.2 Congestion mechanism

The congestion mechanism which plays an essential role to control congestion lengths is described quantitatively based on the volume balance at each signalized intersection given by Eq. (1).

\[ \begin{align*}
\text{i) The congestion at each signalized intersection occurs at the time when the excess incoming volume becomes greater than zero, i.e.} & \\
& x_i(j, m, k - 1) = 0 \quad \text{and} \quad x_i(j, m, k) > x_i(j, m, k) \\
\text{ii) It disappears at the time when the excess incoming volume becomes less than or equal zero, i.e.} & \\
& x_i(j, m, k - 1) > 0 \quad \text{and} \quad x_i(j, m, k) + x_i(j, m, k) \leq x_i(j, m, k) \\
\text{iii) It continues so long as the excess incoming volume} & \\
& x_i(j, m, k - 1) > 0 \quad \text{and} \quad x_i(j, m, k) + x_i(j, m, k) > x_i(j, m, k)
\end{align*} \]

The signal control system is synthesized using the feedback control at each signalized intersection (see Fig. 4). The purpose of the signal control system of two-way traffic arterials is to find such control input that it makes the following performance criterion \( J_a(k) \) minimize

\[ J_a(k) = \sum_{j=1}^{N} \sum_{m=1}^{M} g_j(j, m, k) \]

where \( N \) denotes the number of signalized intersections along the direction for \( j \), the function \( g(j, m, k) \) and the control error \( e(j, m, k) \) are defined by using reference input \( l(j, m, k) \).

Fig. 1 Two-way traffic flows along arterial.

Fig. 2 Volume balance for each lane at each signalized intersection.

incoming volume and outgoing volume (see Fig. 2). \( \psi_j(j, m, k) \) denotes the net traffic flow rate and \( \xi_j(j, m, k) \) is evaluated by taking the value of dividing \( x_i(j, m, k) \) by \( \psi_j(j, m, k) \) under any traffic flow conditions.

\[ \begin{align*}
\psi_j(j, m, k) & = x_i(j, m, k) - x_o(j, m, k) \\
\xi_j(j, m, k) & = \frac{x_i(j, m, k)}{\psi_j(j, m, k)} \\
& \text{for any traffic flow conditions.}
\end{align*} \]

3. SIGNAL CONTROL SYSTEM

In the volume balance at each signalized intersection, it is assumed that the incoming volume \( x_i(j, m, k) \) is measured, and the outgoing volume \( x_o(j, m, k) \) is controlled by the three signal control parameters at the signalized intersection concerned. As the result, the outgoing volume \( x_o(j, m, k) \) is replaced by the control input \( u(j, m, k) \).

The signal control system is then written by

\[ \begin{align*}
x_i(j, m, k) & = x_i(j, m, k - 1) + x_i(j, m, k) + x_i(j, m, k) \\
y_i(j, m, k) & = l_s(j, m, k) - x_i(j, m, k)
\end{align*} \]

where the upper limit of the control input is determined by Eq. (2) and shown in Fig. 3. The observation equation of the congestion length \( y_c(j, m, k) \) is described in such a way that the state variable is multiplied by a "transformation factor" \( l_{cm}(j, m, k) \).

The signal control system of the congestion length is considered on the arterials; in this control system, the reference input, control input and output are given by the permitted congestion length \( l(j, m, k) \), the three signal control parameters and the congestion length respectively.

In this way, the signal control system of two-way traffic arterials is to find such control input that it makes the following performance criterion \( J_a(k) \) minimize

\[ J_a(k) = \sum_{j=1}^{N} \sum_{m=1}^{M} g(j, m, k) \]

where \( N \) denotes the number of signalized intersections along the direction for \( j \), the function \( g(j, m, k) \) and the control error \( e(j, m, k) \) are defined by using reference input \( l(j, m, k) \).

\[ g(j, m, k) = \begin{cases} 0 & e(j, m, k) \geq 0 \\ |e(j, m, k)| & e(j, m, k) < 0 \end{cases} \]

Fig. 3 Characteristic of control input.
Step 3. The incoming volumes of congestion cases are recalculated by

\[ x_i^{(o)}(j, m, k) = x_i^{(o)}(j, m, k) + x_i^{(o)}(j, m, k - 1) \]
\[ x_s^{(o)}(j, m, k) = x_s^{(o)}(j, m, k) + x_s^{(o)}(j, m, k - 1) \]
\[ x_u^{(o)}(j, m, k) = x_u^{(o)}(j, m, k) + x_u^{(o)}(j, m, k - 1) \]
\[ x_{ol}^{(o)}(j, m, k) = x_{ol}^{(o)}(j, m, k) + x_{ol}^{(o)}(j, m, k - 1) \]
\[ x_{ul}^{(o)}(j, m, k) = x_{ul}^{(o)}(j, m, k) + x_{ul}^{(o)}(j, m, k - 1) \]
\[ x_{or}^{(o)}(j, m, k) = x_{or}^{(o)}(j, m, k) + x_{or}^{(o)}(j, m, k - 1) \]
\[ x_{ur}^{(o)}(j, m, k) = x_{ur}^{(o)}(j, m, k) + x_{ur}^{(o)}(j, m, k - 1) \]
\[ x_{ol}^{(o)}(j, m, k) = x_{ol}^{(o)}(j, m, k) + x_{ol}^{(o)}(j, m, k - 1) \]
\[ x_{ur}^{(o)}(j, m, k) = x_{ur}^{(o)}(j, m, k) + x_{ur}^{(o)}(j, m, k - 1) \]

where \( x_i^{(o)}(j, m, k) \), \( x_s^{(o)}(j, m, k) \), \( x_u^{(o)}(j, m, k) \), \( x_{ol}^{(o)}(j, m, k) \) are incoming volume of left-turn-, straightforward-, right-turn- and straightforward-left-turn-lanes respectively.

Step 4. The net traffic flow rate for each lane at the signalized intersection is evaluated by

\[
\psi_{ol}^{(o)}(j, m, k) = r_{ol}^{(o)}(j, m, k) \cdot c_{ol}(j, m, k) \\
\psi_{ul}^{(o)}(j, m, k) = r_{ul}^{(o)}(j, m, k) \cdot c_{ul}(j, m, k) \\
\psi_{or}^{(o)}(j, m, k) = r_{or}^{(o)}(j, m, k) \cdot c_{or}(j, m, k) \\
\psi_{ur}^{(o)}(j, m, k) = r_{ur}^{(o)}(j, m, k) \cdot c_{ur}(j, m, k) \\
\psi_{ol}^{(o)}(j, m, k) = r_{ol}^{(o)}(j, m, k) \cdot c_{ol}(j, m, k) \\
\]

where \( c_{ol}(j, m, k) \), \( c_{ul}(j, m, k) \), \( c_{or}(j, m, k) \), \( c_{ur}(j, m, k) \) are the capacities and \( r_{ol}^{(o)}(j, m, k) \), \( r_{ul}^{(o)}(j, m, k) \), \( r_{or}^{(o)}(j, m, k) \), \( r_{ur}^{(o)}(j, m, k) \) are the green splits, of left-turn-, straightforward-, right-turn- and straightforward-left-turn-lanes respectively.

Step 5. The green time of each lane are evaluated by multiplying the cycle length by the green split.

Step 6. The excess incoming volume \( x_i^{(o)}(j, m, k) \) is evaluated based on the volume balance

\[
x_i^{(o)}(j, m, k) = x_s^{(o)}(j, m, k) - x_{ol}^{(o)}(j, m, k) \\
\]

\[
\begin{align*}
\mathcal{J}_s^{(o)}(j, m, k) &= \psi_{ol}^{(o)}(j, m, k) \cdot \psi_{ul}^{(o)}(j, m, k) \\
&\quad - \psi_{or}^{(o)}(j, m, k) \cdot \psi_{ur}^{(o)}(j, m, k) \\
&\quad - \psi_{ol}^{(o)}(j, m, k) \cdot \psi_{ul}^{(o)}(j, m, k) \\
&\quad + \psi_{or}^{(o)}(j, m, k) \cdot \psi_{ur}^{(o)}(j, m, k) \\
&\quad - \psi_{ol}^{(o)}(j, m, k) \cdot \psi_{or}^{(o)}(j, m, k) \\
&\quad + \psi_{ul}^{(o)}(j, m, k) \cdot \psi_{ur}^{(o)}(j, m, k) \\
&\quad - \psi_{ol}^{(o)}(j, m, k) \cdot \psi_{ul}^{(o)}(j, m, k) \\
&\quad + \psi_{or}^{(o)}(j, m, k) \cdot \psi_{ur}^{(o)}(j, m, k) \\
&\quad - \psi_{ol}^{(o)}(j, m, k) \cdot \psi_{or}^{(o)}(j, m, k) \\
&\quad + \psi_{ul}^{(o)}(j, m, k) \cdot \psi_{ur}^{(o)}(j, m, k) \\
\end{align*}
\]
executed sequentially from limits by their increments. This control algorithm is
are corrected starting from the lower limits to the upper
\[ \begin{align*}
\dot{y}_e^{(n)}(j,m,k) &= \dot{y}_e^{(0)}(j,m,k) \\
\dot{y}_a^{(n)}(j,m,k) &= \dot{y}_a^{(0)}(j,m,k) \\
\dot{y}_c^{(n)}(j,m,k) &= \dot{y}_c^{(0)}(j,m,k) \\
\dot{y}_v^{(n)}(j,m,k) &= \dot{y}_v^{(0)}(j,m,k)
\end{align*} \]  
(20)

where \( \dot{y}_e^{(n)}(j,m,k) \), \( \dot{y}_a^{(n)}(j,m,k) \), \( \dot{y}_c^{(n)}(j,m,k) \),
\( \dot{y}_v^{(n)}(j,m,k) \) are congestion length, of left-turn-, straightforward-, right-turn- and straightforward-
left-turn-lanes respectively.

Step 8. The green time and green splits are evaluated for all
approaches based on the phase at each signalized intersection.

Step 9. If the following control index
\[ \max \| \dot{E}^{(1)}(j,1,k) \| \| \dot{E}^{(2)}(j,2,k) \| \| \dot{E}^{(3)}(j,3,k) \| \leq \varepsilon 
\]  
(21)
\( \varepsilon > 0 \)
is satisfied, we apply the green splits and the cycle length at optimum time values and proceed to Step 12.

Step 10. Otherwise
\[ \max \| \dot{E}^{(1)}(j,1,k) \| \| \dot{E}^{(2)}(j,2,k) \| \| \dot{E}^{(3)}(j,3,k) \| > \varepsilon 
\]  
(22)
\( \varepsilon > 0 \)
then the green splits whose approach of vehicles take the maximum control error are corrected using
\[ \begin{align*}
\dot{r}_e^{(n+1)}(j,m,k) &= \dot{r}_e^{(n)}(j,m,k) + \Delta r_e^{(n)}(j,m) \\
\dot{r}_a^{(n+1)}(j,m,k) &= \dot{r}_a^{(n)}(j,m,k) + \Delta r_a^{(n)}(j,m) \\
\dot{r}_c^{(n+1)}(j,m,k) &= \dot{r}_c^{(n)}(j,m,k) + \Delta r_c^{(n)}(j,m) \\
\dot{r}_v^{(n+1)}(j,m,k) &= \dot{r}_v^{(n)}(j,m,k) + \Delta r_v^{(n)}(j,m)
\end{align*} \]  
(23)

If, \( \dot{r}_e^{(n+1)}(j,m,k) > r_{e,\text{max}} \) then proceed to Step 11.
If, \( \dot{r}_e^{(n+1)}(j,m,k) \leq r_{e,\text{max}} \) then return to Step 4.

Step 11. The cycle length is corrected by
\[ \dot{c}_e^{(n+1)}(j,m,k) = \dot{c}_e^{(n)}(j,m,k) + \Delta c_e^{(n)}(j,m) \]  
(24)

If, \( \dot{c}_e^{(n+1)}(j,m,k) > c_{e,\text{max}} \) then proceed to Step 12.
If, \( \dot{c}_e^{(n+1)}(j,m,k) \leq c_{e,\text{max}} \) then return to Step 2.

Step 12. By using the optimal values of the green splits and cycle length, the balance offset which maximizes a through band width along the arterial is evaluated by the Fieser's method.

In this algorithm, the cycle length and the green splits are corrected starting from the lower limits to the upper limits by their increments. This control algorithm is executed sequentially from \( k=1 \) to \( k=k_f \) and from \( j=1 \) to \( j=N \). The hierarchical structure and the performance
criteria of the balance control algorithm are shown in Fig. 5 and Table 1. From the relation of the offset control, the maximum values of cycle lengths at \( N \) signalized intersections are set in common, and the green split and the offset at each signalized intersection are calculated again.

5. SIMULATION RESULTS

The balance control algorithm of the congestion length for two-way traffic arterials is simulated at signalized intersections during evening rush hours in Hiroshima city, Japan (see Fig. 6 to Fig. 9). The parameters and the incoming volumes are arranged for the simulation at signalized intersections based on measurement data. The incoming volumes for each lane vary randomly cycle by cycle at signalized intersections as shown in Fig. 10. The

Fig. 5 Hierarchical structure of balance control algorithm.

Table. 1 Performance criteria of balance control algorithm.

<table>
<thead>
<tr>
<th>Level</th>
<th>Performance criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>First level</td>
<td>Minimize ( J_e(k) ) ( c_e, r_g, t_{off} )</td>
</tr>
<tr>
<td>Second level</td>
<td>Minimize ( J_g(k) ) ( r_g, t_{off} )</td>
</tr>
</tbody>
</table>

Fig. 6 Arterial consisting of three signalized intersections in Hiroshima city.
reference input is set as \( l_j, (j, m, k), =0 \) m for all approaches. In the signal control algorithm, the green time defined by the product of the cycle length and the green split are searched so as to minimize the performance criterion of Eq. (7). Cycle lengths are compared between the simulation value and the measurement value controlled by a pattern selection method at oversaturated signalized intersections in Fig. 11. Cycle lengths are controlled more widely and adaptively by the control algorithm. The values of green time for straightforward vehicles are shown at the same signalized intersection in Fig. 12. The balance offsets by Fieser's method are searched using the optimal values of the cycle length and the green split so as to maximize the through band width (see Fig. 13). As the results, while congestions occur at some signalized intersections in real circumstances, the congestion lengths are controlled so as to become nearly zero at all signalized intersections by the balance control algorithm (see Fig. 14).

Fig. 8 Type and phase at In front of Hiroshima City Office signalized intersection.

Fig. 7 Type and phase at Sumiyoshicho signalized intersection.

Fig. 9 Type and phase at Kokutaiji signalized intersection.
This paper studies the development of a signal control system along the Route 2 in Hiroshima city, Japan from the viewpoint of a digital control for traffic flow dynamics. The following have been shown.

i) A signal control system of the congestion length is described by a nonlinear dynamic system based on the volume balance for each lane at each signalized intersection.

ii) The signal control system is synthesized for two-way traffic arterials using the feedback control.

iii) Optimal signal control parameters of the arterials are searched systematically and sequentially by the balance control algorithm.

iv) From the comparison between measurement values controlled by a pattern selection method and simulation values, it is confirmed that the signal control system and the signal control algorithm work effectively so as to minimize the performance criterion along the arterial.

It is a future problem that the proposed signal control system...
algorithm will be applied for more signalized intersections along the Route 2 in Hiroshima city.

REFERENCES


